The Real Net National Product in Sustainable Development

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ABSTRACT

This paper is related to social welfare and sustainability. The real NNP represents the maximized value of flow of goods and services that are produced by the productive assets of the society. It is important to investigate whether the concept of NNP can serve as an indicator of sustainability. In this paper an attempt has been taken to clarify this with theoretical calculations. The instantaneous increases in real NNP over time are an accurate indicator of true welfare improvements. The paper shows that welfare is increasing instantaneously over time if and only if real NNP is increasing instantaneously over time. It is also shown the relation between the Divisia index of real consumption prices and dynamic welfare evaluation. The paper emphasizes on optimal growth and growth without optimality, and is examined sustainability in these two cases.

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1. INTRODUCTION

Net national product (NNP) is an important item for a country. Martin Weitzman published his seminal paper (Weitzman 1976) on the significance for dynamic welfare of comprehensive national accounting aggregates, where he had included important theoretical contributions on welfare and sustainable accounting. Weitzman (1976), Solow (1986), Hartwick (1990), Mäler (1991) and some other economists laid the foundation for a concept of NNP, which is adjusted for the depletion of natural and environmental resources. In the aftermath of the World Commission on Environment and Development (WCED 1987), it became important to investigate whether the concept of NNP can serve as an indicator of sustainability. In this paper we also emphasis on social welfare comparisons based on national accounting aggregates, in the tradition of Weitzman (1976). If NNP of a society increases usually it is thought that the society is in better position but according to green NNP it is an apparent concept of the society. The NNP represents the maximized value of flow of goods and services that are produced by the productive assets of the society. It is known that the current-value Hamiltonian of an optimal growth problem represents in welfare terms the level of stationary-equivalent future utility. But it is also apparent that a current-value Hamiltonian is essentially comprehensive NNP expressed in utility units, and monetary units are comparatively better. Asheim (2010) emphasize on

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per capita welfare to normative foundation for transfers between different economies. He shows how the theory of national accounting can be used for global welfare comparisons, under some given assumptions. Fleurbaey and Gaulier (2009) introduce a careful welfare-economic analysis of how to correct per capita GDP for labor, risk of unemployment, health, household demography, inequalities and sustainability. But they limit their analysis welfare comparisons to the hypothetical case of a population permanently exposed to the current conditions. Asheim and Wei (2009) show various sectors' national income into national savings which is compatible with an important line of theoretical literature on comprehensive national accounting. Asheim (2011) discusses the literature on welfare comparisons based on national accounting aggregates which have primarily been concerned with developing and applying the theory of national accounting to the welfare comparison within economies. We have introduced some propositions with proof related to the discussion where appropriate. Mohajan (2011) analyzes the theory of green national accounting and, emphasizes on social welfare and sustainable accounting. He also shows that green net national product measures the gross social profit rather than net social profit.

The paper is organized as follows: In section-2 the notations and related definitions are introduced which are used to discuss sections-3 and -4. The optimal growth of the society is developed in section-3, where it is shown that sustainability is possible for optimal growth. In section-4 we discuss economic growth without optimality which shows that sustainable development is not possible without optimality. Section-5 is of concluding remarks.

2 THE NOTATIONS AND RELATED DEFINITIONS

Assume that population is constant and the current instantaneous well-being at time *t* depends on the vector of commodities $\mathbf{C}(t) = (C_1(t),...,C_m(t))$ consume at time *t*. The vector of capital stocks at time *t* be $\mathbf{K}(t) = (K_1(t),...,K_n(t))$ which indicates not only different kinds of man-made capital, but also stock of natural capital, environmental assets, human capital and other durable productive assets. For any vector of capital stocks **K** at time *t*, the resource allocation mechanism determines the consumption and investment flows. The investment flows indicates the development of the capital stocks. The vector of net investment at time *t* be $\mathbf{I}(t) = (I_1(t),...,I_n(t)) = \dot{\mathbf{K}}(t) \geq 0$. The net investment flow of a natural capital asset is negative if the overall extraction rate exceeds the replacement rate. The NNP is the maximized market value of current productive capacity in a perfect market economy where **C** and **I** are included in NNP and valued at market prices. Since NNP is used for consumption now and accumulation of capital goods yields increased future consumption, so that to relate NNP growth to welfare improvement requires a notion of dynamic welfare. The consumption $\mathbf{C}(t) \geq 0$ generates utility $u(t) = u(\mathbf{C}(t))$, where *u* is a time-invariant strictly increasing , concave and differentiable function, which indicates the instantaneous well-being at time *t*. Utility $u(\mathbf{C}(t))$ is derived from the vector of the consumption flows.

3. OPTIMAL GROWTH IN THE SOCIETY

For any consumption-flow $\{\mathbf{C}(t)\}$, overall inter-temporal welfare is measured by;

$$W\{\mathbf{C}(t)\} = \int_0^\infty u(\mathbf{C}(t))e^{-\delta t}\, dt \,. \tag{1}$$

where δ is positive and constant utility discount rate. We assume that the resource allocation mechanism, for any vector of initial capital stocks $\mathbf{K}_0$, implements a path $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}$ which maximizes (1) over all feasible consumption paths. The maximum principle expresses that there exists a path $\{\mathbf{\Psi}(t)\}$ of investment prices in terms of utility such that $(\mathbf{C}^*(t), \mathbf{I}^*(t))$ maximizes $u\,\mathbf{C}(t)+\mathbf{\Psi}(t)\mathbf{I}(t)$. Associate welfare $W(\mathbf{K}_0)$ with the utility level which is constant, is equally as good as the implemented path;

$$W^*(\mathbf{K}_0)=\frac{\int_0^\infty u(\mathbf{C}^*(t))e^{-\delta t}\,dt}{\int_0^\infty e^{-\delta t}\,dt}=\delta\int_0^\infty u(\mathbf{C}^*(t))e^{-\delta t}\,dt\,, \tag{2}$$

where $\int_0^\infty e^{-\delta t}\,dt=\frac{1}{\delta}$. A main result of Weitzman (1976) is as follows:

$$u\,\mathbf{C}^*(0)+\mathbf{\Psi}(0)\mathbf{I}^*(0)=\delta\int_0^\infty u(\mathbf{C}^*(t))e^{-\delta t}\,dt\,. \tag{3}$$

Hence from (2) and (3) under discounted utilitarianism we get;

$$W^*(\mathbf{K}_0)=u\,\mathbf{C}^*(0)+\mathbf{\Psi}(0)\mathbf{I}^*(0). \tag{4}$$

We know that $\mathbf{\Psi}(0)$ is the vector of partial derivatives of $\int_0^\infty u(\mathbf{C}^*(t))e^{-\delta t}\,dt$ with respect to the initial stocks $\mathbf{K}_0$, we obtain that the vector of partial derivatives of W, $\nabla W^*(\mathbf{K}_0)$ is equal to $\delta\,\mathbf{\Psi}(0)$, where ∇ denotes a vector of partial derivatives. Hence,

$$\nabla W^*(\mathbf{K}_0)=\delta\,\mathbf{\Psi}(0)\Rightarrow\ \delta\,u(\mathbf{C})+\nabla W^*(\mathbf{K}_0)\mathbf{I}=\delta\left(u(\mathrm{C})+\mathbf{\Psi}(0)\mathbf{I}\right)\ \text{for } \delta>0.$$

So that maximum principle now becomes;

$$(\mathbf{C}^*(0), \mathbf{I}^*(0))\ \text{maximizes}\ \delta\,u(\mathbf{C})+\nabla W^*(\mathbf{K}_0)\mathbf{I}\,.$$

Social preferences are represented by $\inf_t\ u(\mathbf{C}(t))$. Assume that the resource allocation mechanism implements maximin which leads to an efficient path with constant utility. Burmeister and Hammond (1977), and Dixit, Hammond and Hoel (1980) called this a regular maximin path (the ranking of paths according to the utility of the worst-off generation). With the initial condition $\mathbf{K}^*(0)=\mathbf{K}_0$, there exists a path of utility discount factors $\{\mu(t)\}$ such that the implemented path $\{\mathbf{C}^*(t),\mathbf{I}^*(t),\mathbf{K}^*(t)\}$ maximizes $\int_0^\infty \mu(t)u(\mathbf{C}^*(t))\,dt$

over all feasible consumption paths. Since $u(\mathbf{C}^*(t))$ is constant, so that $\int_0^\infty \mu(t)\,dt$ is finite. This condition holds if the supporting utility discount rates $\left(-\frac{\dot{\mu}(t)}{\mu(t)}\right)$, are positive and do not decrease too fast. Again, by the maximum principle, there exists a path $\{\mathbf{\Psi}(t)\}$ of investment prices in terms of utility such that $(\mathbf{C}^*(t), \mathbf{I}^*(t))$ maximizes $u(\mathbf{C})+\ \mathbf{\Psi}(t)\mathbf{I}$. Associate welfare $W(\mathbf{K}_0)$ also in this case with the utility level which is constant , is equally as good as the implemented path:

$$W^*(\mathbf{K}_0) = u\left(\mathbf{C}^*(t)\right) = \frac{\int_0^\infty \mu(t)\,u\left(\mathbf{C}^*(t)\right)dt}{\int_0^\infty \mu(t)\,dt}. \tag{5}$$

By the converse of Hartwick's rule, it can be expressed that $\mathbf{C}(t)\,\mathbf{I}^*(t) = 0$ at each t (Hartwick 1977, Dixit, Hammond and Hoel 1980, Withagen and Asheim 1998). As a result equation (4) remains unchanged according to the condition maximin. As before $\mathbf{\Psi}(0)$ is the vector of partial derivatives of $\int_0^\infty \frac{\mu(\mathrm{t})}{\mu(0)} u\left(\mathbf{C}^*(t)\right)dt$ with respect to the initial stocks $\mathbf{K}_0$, by invoking the envelope theorem, we obtain that the vector of partial derivatives of W, $\nabla W(\mathbf{K}_0)$ is equal to $\delta^*\ \mathbf{\Psi}(0)$ where

$$\delta^* = \frac{\mu(0)}{\int_0^\infty \mu(\mathrm{t})\,dt}$$

is the infinitely long term supporting utility discount rate at time t = 0. So that maximum principle now becomes;

$(\mathbf{C}^*(0), \mathbf{I}^*(0))$ maximizes $\delta^*\ u(\mathbf{C}) + \nabla W(\mathbf{K}_0)\mathbf{I}$, for $\delta^* > 0$.

For maximization we get;

$$\begin{aligned} &\delta^*\ u\left(\mathbf{C}^*(0)\right) + \nabla W(\mathbf{K}_0)\mathbf{I}^*(0) \\ &= \delta^*\ u\left(\mathbf{C}^*(0)\right) + \delta^*\ \boldsymbol{\psi}(0)\mathbf{I}^*(0) \\ &= \delta^*\left(u\left(\mathbf{C}^*(0)\right) + \boldsymbol{\psi}(0)\mathbf{I}^*(0)\right), \text{ where } \delta^* > 0. \end{aligned} \tag{5a}$$

The term $u\,\mathbf{C}^*(0)+\mathbf{\Psi}(0)\mathbf{I}^*(0)$ is the net national product in terms of utility (utility NNP). For both cases discounted utilitarianism and maximin, utility NNP represents dynamic welfare globally; i.e., welfare is greater if and only if utility NNP is greater.

From the above discussion the implementation of an efficient path can be written as follows (Asheim and Buchhold 2004):

Let $\{\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)\}$ *be the path implemented by the resource allocation mechanism with* $\mathbf{K}^*(0) = \mathbf{K}_0$ *is the given initial condition. There exists a continuous path of positive supporting utility discount factors* $\{\mu(t)\}$*, with corresponding discount rates* $\left(-\dfrac{\dot{\mu}(t)}{\mu(t)}\right)$ *being positive at almost every t, such that it is as if* $\{\mathbf{C}^*(t),\mathbf{I}^*(t), \mathbf{K}^*(t)\}$ *maximizes* $\int_0^\infty \mu(t)u\left(\mathbf{C}^*(t)\right)dt$ *over all feasible consumption paths with* $\mathbf{K}^*(0) = \mathbf{K}_0$ *is the given initial condition.*

For optimal growth equation (1) maximizes and maximized welfare at time t can be written as;

$$W^*(t)=\int_t^\infty u\left(\mathbf{C}^*(\tau)\right)e^{-\delta(\tau-t)}\,d\tau\,. \tag{6}$$

Proposition 1: *Utility NNP represents dynamic welfare globally*.

Proof: From the definitions of discounted utilitarianism and maximin it is realized that the term $u\,\mathbf{C}^*(0)+\mathbf{\Psi}(0)\mathbf{I}^*(0)$ is the net national product in terms of utility (utility NNP). The discounted utilitarianism in (4) indicates that utility NNP is maximized and hence dynamic welfare is maximized. Again the maximin in (5a) expresses ($\mathbf{C}^*(0)$, $\mathbf{I}^*(0)$) maximizes $\delta^*\,u(\mathbf{C})+\nabla W(\mathbf{K}_0)\mathbf{I}$ for $\delta^*>0$. As a result utility NNP maximizes, which is the dynamic global welfare. Q.E.D.

Let $\{\mathbf{\Psi}(t)\}$ represents the trajectory of the dual vector of shadow investment prices, relative to utility being the numeraire. The current-value of Hamiltonian can be given by (Asheim and Weitzman 2001):

$$H(\mathbf{C}(t),\mathbf{I}(t);\mathbf{\Psi}(t))=u\,\mathbf{C}(t)+\mathbf{\Psi}(t)\mathbf{I}(t) \text{ for all } t. \tag{7}$$

Now ($\mathbf{C}^*(t)$, $\mathbf{I}^*(t)$) maximizes $H(\mathbf{C}(t),\mathbf{I}(t);\mathbf{\Psi}(t))$ as follows:

$$H^*(t)=\mathrm{H}\,(\,\mathbf{K}^*(t)\,,\mathbf{\Psi}(t)) = \max_{(\mathbf{C}(t),\mathbf{I}(t))\in S(\mathbf{K}^*(t))} u\,\mathbf{C}(t)+\mathbf{\Psi}(t)\mathbf{I}(t) = u\,\mathbf{C}^*(t)+\mathbf{\Psi}(t)\mathbf{I}^*(t). \tag{8}$$

Since $\mathbf{\Psi}(t)\mathbf{I}^*(t)$ is the value of net investments so that the co-state differential equation can be written as (Asheim and Weitzman 2001):

$$\nabla H_{\mathbf{K}}(\mathbf{K}^*(t), \mathbf{\Psi}(t)) = \delta\, \mathbf{\Psi}(t) - \dot{\mathbf{\Psi}}(t). \tag{9}$$

Since $\delta = -\dfrac{\dot{\mu}(t)}{\mu(t)}$ then (9) becomes,

$$\nabla H_{\mathbf{K}}(\mathbf{K}^*(t), \mathbf{\Psi}(t)) = -\frac{\dot{\mu}(t)}{\mu(t)}\; \mathbf{\Psi}(t) - \dot{\mathbf{\Psi}}(t). \tag{9a}$$

Differentiating (8) with respect to *t* and using (9) we get;

$$\dot{H}^*(t) = \nabla H_{\mathbf{K}}\mathbf{I}^*(t) + \nabla H_{\mathbf{\Psi}}\; \dot{\mathbf{\Psi}}(t) = \left(\delta\, \mathbf{\Psi}(t) - \dot{\mathbf{\Psi}}(t)\right)\mathbf{I}^*(t) + \nabla H_{\mathbf{\Psi}}\; \dot{\mathbf{\Psi}}(t). \tag{10}$$

Differentiating right side of (8) with respect to *t* we get;

$$\dot{H}^*(t) = \nabla u\left(\mathbf{C}^*(t)\right)\dot{\mathbf{C}}^*(t) + \frac{d}{dt}\left(\mathbf{\Psi}(t)\,\mathbf{I}^*(t)\right). \tag{11}$$

From (10) and (11) it follows that:

$$\nabla u\left(\mathbf{C}^*(t)\right)\dot{\mathbf{C}}^*(t) + \frac{d}{dt}\left(\mathbf{\Psi}(t)\,\mathbf{I}^*(t)\right) = \delta\, \mathbf{\Psi}(t)\,\mathbf{I}^*(t). \tag{12}$$

Differentiating (6) with respect to *t* we get;

$$\dot{W}^*(t) = -u\left(\mathbf{C}^*(\tau)\right) + \delta \int_t^\infty u\left(\mathbf{C}^*(\tau)\right) e^{-\delta(\tau - t)}\, d\tau .$$

Using the property of integration by parts we get;

$$\dot{W}^*(t) = \int_t^\infty \nabla u\left(\mathbf{C}^*(\tau)\right)\dot{\mathbf{C}}^*(\tau)\, e^{-\delta(\tau - t)}\, d\tau .$$

By using (12) we get;

$$\dot{W}^*(t) = -\int_t^\infty \frac{d}{d\tau}\left(\mathbf{\Psi}(\tau)\,\mathbf{I}^*(\tau)\, e^{-\delta(\tau - t)}\right) d\tau = \mathbf{\Psi}(t)\,\mathbf{I}^*(t). \tag{13}$$

Proposition 2: $\{\mathbf{\Psi}(t)\}$ *be the trajectory of the dual vector of shadow investment prices, relative to utility being the numeraire and* $\mathbf{I}^*(t)$ *is the maximum investment then maximized welfare increases if and only if* $\mathbf{\Psi}(t)\mathbf{I}^*(t) > 0$.

Proof: Suppose welfare increases, that is $\dot{W}^*(t) > 0$. From the definition of shadow prices we can write, $\mathbf{\Psi}(t) > 0$ for each *t*. If the total invest $\mathbf{I}(t)$ is maximized then maximized investment becomes $\mathbf{I}^*(t) > 0$ for each *t*. Accordingly, $\mathbf{\Psi}(t)\mathbf{I}^*(t) > 0$.

Conversely, let $\mathbf{\Psi}(t)\mathbf{I}^*(t) > 0$ then the social development will increase and the society will gain maximum welfare, and maximum welfare will increase continuously. Consequently, $\dot{W}^*(t) > 0$. Q.E.D.

Proposition-2 indicates that maximized welfare is increasing if and only if $\mathbf{\Psi}(t)\mathbf{I}^*(t) > 0$. Moreover it indicates that welfare is increasing if and only if measurable NNP exceeds the value of consumption, this is a different kind of welfare significance than which was shown by Weitzman (1976), where higher welfare is indicated by higher NNP.

In the Markovian and stationary economy the consumption-investment pair ($\mathbf{C}(t)$, $\mathbf{I}(t)$) at time *t* is determined by the vector of capital stocks at time *t* and does not depend directly on *t*. Hence, if $\{\mathbf{C}(\tau)\}_{\tau=t}^{\infty}$ is the implemented path given the initial stock $\mathbf{K}(t) = \mathbf{K_0}$, then the dynamic welfare of this path becomes;

$$V(\mathbf{K}_0) = \int_t^{\infty} u(\mathbf{C}(\tau)) e^{-\delta(\tau - t)} \, d\tau \,, \tag{14}$$

is a function solely of $\mathbf{K_0}$. In particular,

$$\frac{d}{dt}(V(\mathbf{K}(t))) = \nabla V(\mathbf{K}(t)\mathbf{I}(t)) > 0 \tag{15}$$

means that dynamic welfare is increasing at time *t*. Again the partial derivatives of *V* in accounting prices can be written as (Asheim 2003);

$$\mathbf{q}(t) = \frac{\nabla V(\mathbf{K}(t))}{\lambda(t)} = \frac{\mathbf{\Psi}(t)}{\lambda(t)}, \tag{16}$$

where $\lambda(t)$> 0 is the not-directly-observable marginal utility of current expenditures, which may depend on the ‘quantity of money’ at time *t*. From (16) we can write;

$$\mathbf{q}(t)\mathbf{I}(t) = \frac{\nabla V(\mathbf{K}(t))\mathbf{I}(t)}{\lambda(t)} = \frac{1}{\lambda(t)} \frac{d}{dt}(V(\mathbf{K}(t))) > 0\,. \tag{17}$$

Here $\mathbf{q}(t)\mathbf{I}(t)$ represents the value of net investments, and it is often referred to as the genuine savings indicator (Hamilton 1994). Hence $\mathbf{q}(t)\mathbf{I}(t)$ indicates that dynamic welfare is increasing. Now consider that resource allocation mechanism is Markovian, then by differentiating (14) with respect to time *t* we get;

$$\nabla V(\mathbf{K}(t))\mathbf{I}(t) = \frac{d}{dt}\left(V(\mathbf{K}(t))\right) = \int_t^\infty u(\mathbf{C}(\tau))e^{-\delta(\tau-t)}\,d\tau - u(\mathbf{C}(t)) = \delta V(\mathbf{K}(t)) - u(\mathbf{C}(t)),$$

$$u(\mathbf{C}(t)) + \nabla V(\mathbf{K}(t)\mathbf{I}(t)) = \delta V(\mathbf{K}(t)). \tag{18}$$

Now again differentiating (18) with respect to *t* we get,

$$\nabla(u(\mathbf{C}(t)))\dot{\mathbf{C}}(t) + \frac{d}{dt}\{\nabla V(\mathbf{K}(t)\mathbf{I}(t))\} = \delta\,\nabla V(\mathbf{K}(t))\mathbf{I}(t). \tag{19}$$

Since $u(\mathbf{C}^*(t)) + \mathbf{\Psi}(t)\mathbf{I}^*(t)(\mathbf{K}(t))$ is continuous then using $\delta = -\dfrac{\dot{\mu}(t)}{\mu(t)}$ and $\nabla V(\mathbf{K}(t)) = \mathbf{\Psi}(t)$ we can write (19) as (Dixit, Hammond and Hoel 1980):

$$\nabla(u(\mathbf{C}(t)))\dot{\mathbf{C}}(t) + \frac{d}{dt}(\mathbf{\Psi}(\mathrm{t})\mathbf{I}(t)) = -\frac{\dot{\mu}(t)}{\mu(t)}\mathbf{\Psi}(\mathrm{t})\mathbf{I}(t). \tag{19a}$$

Equation (19a) indicates that the change in utility NNP equals the supporting utility discount rate times the value of net investments. Now consider the Lagrange multiplier $\delta(\mathbf{K}) > 0$ on the lower bound for utility which can be interpreted as a supporting utility discount rate. Then for every K, we can write $(\mathbf{C}(\mathbf{K}), \mathbf{I}(\mathbf{K}))$ maximizes $\delta(\mathbf{K})u(C) + \mathbf{\Psi}(t)$, which is called no waste of welfare improvement. The consumption prices in terms of the numeraire can be written as (Asheim 2003);

$$\mathbf{p}(t) = \frac{\nabla u(\mathbf{C}^*(t))}{\lambda(t)}. \tag{20}$$

Using (20) in (19) we get;

$$\lambda(t)\mathbf{p}(t)\dot{\mathbf{C}}(t) + \frac{d}{dt}\{\lambda(t)(\mathbf{q}(t)\mathbf{I}(t))\} = \delta\lambda(t)(\mathbf{q}(t)\mathbf{I}(t)),$$

$$\lambda(t)\mathbf{p}(t)\dot{\mathbf{C}}(t) + \lambda(t)\frac{d}{dt}\{(\mathbf{q}(t)\mathbf{I}(t))\} + \mathbf{q}(t)\mathbf{I}(t)\dot{\lambda}(t) = \delta\lambda(t)(\mathbf{q}(t)\mathbf{I}(t)),$$

$$\mathbf{p}(t)\dot{\mathbf{C}}(t)+\frac{d}{dt}\{(\mathbf{q}(t)\mathbf{I}(t))\}+\frac{\dot{\lambda}(t)}{\lambda(t)}\mathbf{q}(t)\mathbf{I}(t)=\delta(\mathbf{q}(t)\mathbf{I}(t)),$$

$$\mathbf{p}(t)\dot{\mathbf{C}}(t)+\frac{d}{dt}\{(\mathbf{q}(t)\mathbf{I}(t))\}=\left(\delta-\frac{\dot{\lambda}(t)}{\lambda(t)}\right)(\mathbf{q}(t)\mathbf{I}(t)),$$

$$\mathbf{p}(t)\dot{\mathbf{C}}(t)+\frac{d}{dt}\{(\mathbf{q}(t)\mathbf{I}(t))\}=r(t)(\mathbf{q}(t)\mathbf{I}(t)), \tag{21}$$

where

$$r=\delta-\frac{\dot{\lambda}(t)}{\lambda(t)} \tag{22}$$

is the nominal interest rate at time t. Again (9) implies that;

$$\nabla\, \mathrm{H}_{\mathbf{K}}=\delta\,\mathbf{\Psi}(t)-\dot{\mathbf{\Psi}}(t)\;=\;\delta\,\lambda(t)\mathbf{q}(t)-\left(\dot{\lambda}(t)\mathbf{q}(t)+\lambda(t)\dot{\mathbf{q}}(t)\right).$$

Using (22) we get;

$$\nabla\, \mathrm{H}_{\mathbf{K}}\;=\;\lambda(t)\left(r(t)+\frac{\dot{\lambda}(t)}{\lambda(t)}\right)\mathbf{q}(t)-\left(\dot{\lambda}(t)\mathbf{q}(t)+\lambda(t)\dot{\mathbf{q}}(t)\right)$$

$$=\lambda(t)\,r(t)\,\mathbf{q}(t)-\lambda(t)\,\dot{\mathbf{q}}(t). \tag{23}$$

Combining (12) with (23) we can write (21) for maximization as follows;

$$\mathbf{p}(t)\dot{\mathbf{C}}^{*}(t)+\frac{d}{dt}\left\{\left(\mathbf{q}(t)\mathbf{I}^{*}(t)\right)\right\}=r(t)\left(\mathbf{q}(t)\mathbf{I}^{*}(t)\right). \tag{24}$$

The comprehensive NNP in nominal prices, $n(t)$, is the sum of nominal value of consumption and the nominal value of net investment as follows (Asheim and Weitzman 2001):

$$n(t)=\mathbf{p}(t)\mathbf{C}^{*}(t)+\mathbf{q}(t)\mathbf{I}^{*}(t). \tag{25}$$

Since the level of NNP in nominal prices at t depends on arbitrary $\lambda(t)$, so that $\dot{\lambda}(t)>0$ cannot signify welfare improvement. From (13) we get maximized welfare is increasing if and only if NNP exceeds the value of consumption i.e.,

$$\dot{W}^{*}(t)>0 \Leftrightarrow\; n(t)-\mathbf{p}(t)\mathbf{C}^{*}(t)=\mathbf{q}(t)\mathbf{I}^{*}(t)>0.$$

Hence for a change in NNP to indicate a change in welfare, NNP must be measured in real prices.

By the application of price index $\{\pi(t)\}$ nominal prices $\{\mathbf{p}(t),\mathbf{q}(t)\}$ turns into real prices $\{\mathbf{P}(t),\mathbf{Q}(t)\}$ as follows (Asheim and Weitzman 2001):

$$\mathbf{P}(t)=\frac{\mathbf{p}(t)}{\pi(t)},$$

$$\mathbf{Q}(t)=\frac{\mathbf{q}(t)}{\pi(t)}.$$

Then the real interest rate, $R(t)$ at time *t* in terms of nominal interest rate, *r*(*t*) of (22) is given by:

$$R(t)=r(t)-\frac{\dot{\pi}(t)}{\pi(t)}. \tag{26}$$

A Divisia price index satisfies $\dot{\mathbf{P}}(t)\mathbf{C}^*(t)=0$. Hence we can write,

$$\dot{\mathbf{P}}(t)\mathbf{C}^*(t)=\frac{d}{dt}\left(\frac{p}{\pi}\right)\mathbf{C}^*(t)=\frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)-\dot{\pi}(t)\mathbf{p}(t)\mathbf{C}^*(t)}{\pi^2}=0.$$

$$\text{i.e., } \frac{\dot{\pi}(t)}{\pi(t)}=\frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t)}. \tag{27}$$

Now define comprehensive NNP in real Divisia prices, *N*(*t*), as the sum of the real value of net investments from (25) as follows:

$$N(t)=\mathbf{P}(t)\mathbf{C}^*(t)+\mathbf{Q}(t)\mathbf{I}^*(t). \tag{28}$$

Differentiating (28) and then using (24) we get;

$$\dot{N}(t)=\dot{\mathbf{P}}(t)\mathbf{C}^*(t)+\mathbf{P}(t)\dot{\mathbf{C}}^*(t)+\frac{d}{dt}(\mathbf{Q}(t)\mathbf{I}(t))$$

$$=\mathbf{P}(t)\dot{\mathbf{C}}^*(t)+\frac{d}{dt}(\mathbf{Q}(t)\mathbf{I}(t))$$

$$=R(t)\mathbf{Q}(t)\mathbf{I}(t). \tag{29}$$

Since $Q(t)$ is proportional to $\nabla V(\mathbf{K}(t))$, then (29) implies that $\dot{N}(t) > 0$, and hence it indicates the welfare is improving in the society.

Proposition 3: *For the increase of welfare NNP must be measured in real prices.*

Proof: If NNP is measured by nominal prices then (25) indicates that comprehensive NNP, $n(t)$ is the sum of nominal value of consumption and the nominal value of investment. But NNP in nominal prices at t depends on arbitrary $\lambda(t)$, that is why $\dot{n}(t) > 0$ can not signify welfare improvement. Again (29) indicates that real Divisia prices $\dot{N}(t) > 0$ is measured in terms of real prices $P(t)$ and $Q(t)$, where $P(t) > 0$, $Q(t) > 0$, $\mathbf{I}(t) > 0$ and real interest rate $R(t) > 0$, hence $\dot{N}(t) > 0$. Consequently, $\dot{W}(t) > 0$. Q. E.D.

The development is sustainable at the current time, if the utility derived from the current vector of consumption flows can potentially be sustained forever. Assume that stationary technology is combined with optimality, and that the social preferences take sustainability into account, e.g., through the constraint that, at any time, current utility should not exceed the maximum sustainable utility level given the current capital stocks. By optimality, the agents in society expect that development will indeed be sustainable, and these expectations will be reflected by the relative investment prices. In such circumstances, non-decreasing welfare may well correspond to development being sustainable. Hence a non-negative value of net investments (or equivalently, non-negative rate of real NNP growth) may serve as an exact indicator of sustainability (Asheim 2003).

Pezzey (2002) established a one-sided sustainability test under stationary technology, optimality and discount utilitarianism as follows: It is a necessary condition for sustainable development that the value of net investments (or, equivalently, real NNP growth) is non-negative. If stationary technology, optimality and discount utilitarianism are assumed then it follows from (18) that $u(\mathbf{C}(t)) + \nabla V(\mathbf{K}(t)\mathbf{I}(t))$ is a Hicks (1946)–Weitzman (1976) stationary equivalent of future utility. Integrating (18) we get;

$$\int_t^\infty \left(u(\mathbf{C}(t)) + \nabla V(\mathbf{K}(t)\mathbf{I}(t))\right) e^{-\delta(\tau - t)}\, d\tau = V(\mathbf{K}(t)), \text{ since } \int_t^\infty e^{-\delta(\tau - t)}\, d\tau = \frac{1}{\delta}. \tag{30}$$

If optimality is added then,

$$V(\mathbf{K}(t)) \geq \int_t^\infty \bar{u}\ e^{-\delta(\tau - t)}\, d\tau\,, \tag{31}$$

where $\bar{u}$ is the maximum level of utility that can be sustained forever on time t for the given initial stocks at time t. Equations (30) and (31) state that $u(\mathbf{C}(t))$ exceeds the maximum sustainable level at time t if,

$$\mathbf{q}(t)\mathbf{I}(t) = \frac{\nabla V(\mathbf{K}(t))\mathbf{I}(t)}{\lambda(t)} < 0 \Rightarrow \dot{N}(t) < 0.$$

Hence the current level of utility cannot be sustained forever if the value of net investments is negative ($\mathbf{qI} < 0$), or if growth in real NNP is negative ($\dot{N}(t) < 0$), (Pezzey 2002).

Let us consider a society where traditional growth is promoted through high investment in reproducible capital goods, but where lack of pricing of natural capital leads to depletion of natural and environmental resources which is excessive both from the perspective of short-run efficiency and long-run sustainability. At this situation utility growth in the short to intermediate run will lead to current growth in dynamic welfare, where δ is large enough. In this case both the value of net investments and real NNP growth will be positive. The resource depletion may seriously undermine the long-run livelihood of future generations, so that current utility far exceeds the level that can be sustained forever (Asheim 2003).

4. GROWTH WITHOUT OPTIMALITY

Let us consider the global welfare comparisons (Asheim 2003),

- either in one society over time, where $\mathbf{K}' = \mathbf{K}(t')$ is the vector of capital stocks at time t' and $\mathbf{K}'' = \mathbf{K}(t'')$ is the vector of capital stocks at time t'',
- or across different societies, where $\mathbf{K}'$ is the vector of capital stocks in the one society and $\mathbf{K}''$ is the vector of capital stocks in the other society.

Under stationary technology and discount utilitarianism we get from (14);

$$V(\mathbf{K}'') - V(\mathbf{K}') = \int_{\mathbf{K}'}^{\mathbf{K}''} \nabla V(\mathbf{K})\, d\mathbf{K}\,, \tag{32}$$

is a measure of welfare differences which is independent of the path between $\mathbf{K}'$ and $\mathbf{K}''$. Consider that the factor of proportionality equal to one, so that $\mathbf{P} = \nabla u(C)$ and $\mathbf{Q} = \nabla V(\mathbf{K})$, and implying that

$$u(\mathbf{C}) = \nabla u(\mathbf{C})\mathbf{C} = \mathbf{PC}\,. \tag{33}$$

Using (33) we can write (18) as follows:

$$Y = \mathbf{PC} + \mathbf{QI} = \delta\, V(\mathbf{K}) \tag{34}$$

i.e., real NNP = real interest rate × the present value of future consumption,

which is Weitzman's (1976) main result, which established without invoking optimality, but assuming that the vector of partial derivatives of V can be calculated. Now equation (32) becomes;

$$V(\mathbf{K}'') - V(\mathbf{K}') = \int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}\, d\mathbf{K}\,. \tag{35}$$

Here $\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}\, d\mathbf{K}$ is not a difference in wealth, but rather a wealth-like magnitude, to use Samuelson's (1961) term. Equation (35) expresses that a positive welfare difference can be indicated by a positive real value of stock differences ($\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}\, d\mathbf{K} > 0$) or by a positive difference in real NNP ($N'' - N' > 0$), (Asheim 2003).

Proposition 4: *Without optimality under stationary technology and discounted utilitarianism it is possible to obtain a positive welfare difference by real NNP but it is not possible by nominal NNP.*

Proof: In nominal NNP, $n(t)$ is a function of arbitrary $\lambda(t)$, so that a positive difference can not be possible always, so that $(n'' - n')$ can not be always positive. Equation (35) indicates that $\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}\, d\mathbf{K}$ is not a difference in wealth, but rather a wealth-like magnitude, to use Samuelson's (1961) term. But a positive welfare difference can be indicated by a positive real value of stock differences ($\int_{\mathbf{K}'}^{\mathbf{K}''} \mathbf{Q}\, d\mathbf{K} > 0$). Hence a positive welfare difference can be indicated by a positive difference in real NNP, $N'' - N' > 0$. Q.E.D.

Proposition-4 indicates that if stationary technology and discounted utilitarianism hold but optimality does not hold, then the value of net investments and real NNP growth are quite unreliable indicators of sustainability.

It is shown in this paper that welfare stock improvements can be indicated by real NNP flow changes locally in time. However, unless $N(t)$ is monotone between t' and t'', it does not necessarily follow that $N(t') < N(t'')$ indicates that welfare stock is higher at time t'' when compared to an earlier point in time, t'. Because the consumption bundle used as weights in a Divisia price index changes continuously over time.

5. CONCLUDING REMARKS

In this paper we have shown the growth with optimality and the growth without optimality and have examined the sustainability in each case. As like Weitzman (1976) we have assumed that there is no technological progress. The utility NNP represents dynamic welfare globally. The real NNP represents the maximized value of flow of goods and services that are produced by the productive assets of the society. We have shown the relation between the Divisia index of real consumption prices and dynamic welfare evaluation following Asheim and Weitzman (2001). We emphasize on the sustainability and welfare throughout the paper. We have introduced some propositions with proof to make the paper easier to understand and hope that readers take these as genially. In this article we have considered a society where traditional growth is promoted through high investment in reproducible capital goods, but where lack of pricing of natural capital leads to depletion of natural and environmental resources which is excessive both from the perspective of short-run efficiency and long-run sustainability. At this situation utility growth in the short to intermediate run will lead to current growth in dynamic welfare. In such case both the value of net investments and real NNP growth will be positive.

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